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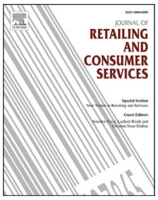

Risk management for barter exchange policy under retail industry

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ABSTRACT

Nowadays, barter exchange has become growingly popular in the national and global industries as an alternative to excessive inventory transfers. Many companies exchange their surplus products on barter platforms for products they need without using money. In a traditional supply chain, the retailer holds unsold products that hinder the supply chain's profitability from reaching its maximum level. This paper solves this issue by proposing a stochastic model of two players (single manufacturer, single retailer) with trade-credit (delayed payment) and barter exchange policy under a supply chain management. In this work, to entice the retailer and increase sales, the manufacturer grants a credit payment facility to the retailer on the items ordered for a specified period, and the manufacturer does not charge any interest on the outstanding amount during this credit period. The concept of a credit period raises the possibility of default risk. In this case, some interest is charged. Several investments are made here to diminish setup and ordering costs and improve the product quality of the system. This work focuses on the flexible production to manage the demand uncertainty and the marginal reduction technology to lessen carbon emissions that occur during the production and inventory holding. In contrast, a retailer can exchange unsold items in the barter market for its required products, which is extensively discussed in this study. Finally, the maximum profit is assessed in terms of credit period, investments, quality improvement, and production rate. The result numerically and graphically proves a huge impact of the barter platform for any business industry on overstock transfers, conserving cash, managing unpredictable demand, and reaching the maximum profit. Moreover, the significant finding is observed in the proposed work that the idea of the flexible production, barter exchange policy, and several investments increase the system profit up to 50.55%.

1. Introduction

Supply chain management (SCM) is a collaboration between players for production planning, resources, information, services, and financial communication, ranging from procurement of raw materials to distribution of scheduled products according to market demand. Several researchers investigate a variety of extensions to the SCM models under different strategies. In reality, the success of any SCM depends on the demand for its products. The demand not only controls the profit or loss of the SCM but also assists it in settling the production time and production rate along with reordering points and safety stock. Generally, demand patterns are of two types: one is deterministic and the other is stochastic. Deterministic demand depends almost entirely on price, inventory, time, service, and quantity and stochastic demand occurs only when SCM is unable to accurately predict consumer demand for its products or services. Zhang et al. (2020) designed a dual-channel SCM with multiple competing retailers, manufacturers, and multiple market demands. In their model, each manufacturer provided products and services via dual channels while each retailer only offered offline services to customers, but they did not consider any strategy to increase sales and manage the overstock situation for the stochastic market

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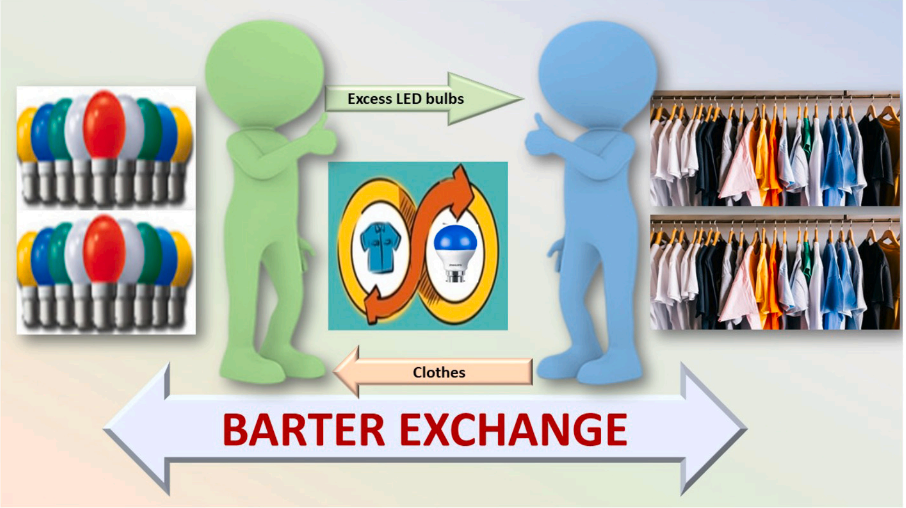

Fig. 1. Barter exchange platform.

demand. A two-tier SCM involving a single manufacturer and retailer under price and generic advertising-based demand was discussed by Ma (2021). In their model, the manufacturer and retailer both invested in generic advertising efforts to increase sales and maximize profits but they did not adopt a trade-credit policy through which the manufacturer could increase order size and the retailer could purchase more products with 0% down payment. A multi-echelon SCM with price-sensitive and stochastic demand was exhibited by Nasiri et al. (2021). In their model, products were delivered to customers through wholesale and retail channels and they allowed different payment methods and proposed an innovative approach for pricing in distribution channels. But they did not focus on flexible production to manage unpredictable demand and barter exchange policy to move overstock inventory. Mishra et al. (2021) discussed an order level inventory model for deteriorating products with backorder under environmental effect. They invested in preservation technology to extend their shelf life and green technology to reduce emissions from the system. However, they did not offer any delayed payment contract to increase sales and the investment to reduce ordering cost. Jena and Meena (2022) developed an omnichannel SCM under test-in-store-and-buy-online (TSBO) and product return policy. In their model, the retailer gave customers the option to find and purchase products online, in-store, or an integrated option buy through online and pick up from store (BOPS). Still, they did not give any particular idea of how the retailer sells the excess inventory and maximizes profits. Kim et al. (2023) studied a robust optimization model under an SCM with uncertain demand. Zhang et al. (2022a) discussed a personalized flexible production system, driven by customers' choice under an SCM. They found out the flexible design of production rate for order customization.

It is found that numerous authors discussed different SCMs under stochastic demand. However, for stochastic demand, unsold products are held with high holding costs for the overstock situation, which is disadvantageous to the overall system. Therefore, determining the optimal inventory level amidst the uncertainty of demand and increasing the profitability of the system has become a significant task for the supply chain manager (Sarkar et al., 2020). To solve this issue, this study develops a stochastic SCM in which the retailer considers a barter platform to exchange overstock products at almost the same retail price for the products it needs.

Barter platform is an internet-based business-to-business (B2B) e-marketplace that gradually prospered over the last 20 years. According to a 2011, the total price of products sold on the platform is approximately 12 billion (Keys and Malnight, 2012). Currently, more than 50,000 companies are engaged in barter business to move their overstock inventory (Hua et al., 2020). In a barter exchange platform, any two or more firms can trade their excess merchandise without money through a broker by giving him only a commission of 1%-2% of the market value. Fig. 1 displays the example of a barter exchange platform in which any company or firm can exchange its excess LED bulbs for some needful dresses from a textile company. Considering a barter exchange policy in an SCM is beneficial for any firm to exchange the unsold inventory, develop a brand image, and cope with unpredictable demand. Again, to avoid the obsolescence and looting of overstock products, increase selling and discover newish customers, it is very significant to grant a credit payment policy from the manufacturer's aspects which is not yet been initiated in any SCM model with barter business.

Under this agreement, the manufacturer grants the retailer a permissible delay period to pay the purchase price. The credit payment strategy incorporates two terms: initial payment as a confirmation cost and a delayed payment as an order volume-dependent cost committed to pay within the delayed period (Wang et al., 2021). The manufacturer does not charge any interest if the retailer clears the dues within the delayed period. However, providing a grace period increases sales as well as default risk, which is related to the retailer's credit status. In general, the default risk rate is considered as an exponential expression of delayed period m_d granted by the supplier and is in the form $F(m_d) = 1 - e^{\theta_l m_d}$, where θ_l is the coefficient of default risk (Chen and Teng, 2015). If $m_d = 0$, the default risk rate $F(m_d) = 0$ and if m_d tends to infinity, then the default risk rate $F(m_d)$ becomes 1 which badly affects the business industry. In case of default risk, the manufacturer charges interest for the delayed period (Wang et al., 2021). An SCM model with credit period and default risk is introduced in this article.

The order size is certainly connected with the delayed period, granted by the manufacturer because by allowing a credit period, the manufacturer indirectly gives the retailer an interest-free loan, which helps him reduce capital constraints. Several researchers like (Wu et al., 2014; Li et al., 2018) widely considered the order volume q as an exponential expression of the credit period m_d as: $q(m_d) = B_q e^{\omega m_d}$, where B_q is the basic order volume and ω is the shape parameter for credit sales. The concept of credit period-dependent order is introduced in this article.

Due to the unpredictable demand, it is very emergent for the manufacturer to consider a flexible production rate to control overstock and understock situations. An SCM with flexible production and uncertain demand was designed by Sarkar et al. (2022b). The proposed study ascertains flexible production to cope with the unexpected demand. Again, the supervisors of any industry always try to maximize the system's profits from every aspect. If players make some investments, the total gain of the system can be increased. Ali et al. (2018) made some service and price level investments in an SCM to ignore the demand disruptions price in retail markets and they did not make any investment to reduce the ordering and utilized the Stackelberg game strategy to solve the model. A two-tier SCM with setup and ordering cost reduction was exhibited by Mishra et al. (2020). The proposed work made some investments to reduce the ordering and setup costs of the system. Customer satisfaction is another important issue for any business industry, and it requires improving product quality. Marculetiu et al. (2023) discussed multiple factors which drive a sustainable supply chain. They explained relation between SCM players and their relation involvement to achieve sustainability within a SCM but did not discuss demand uncertainty effect on SCM sustainability.

Another important issue that any SCM should focus on is environmental protection. Several researchers applied different policies to maintain environmental sustainability. A sustainable multi-tier multi-channel SCM with price, greenness, and advertisement-based demand was investigated by Khorshidvand et al. (2021). In their work, the manufacturer produced products with a fixed production rate but the concept of flexible production is more effective than a fixed production rate in a smart production system. Mittal and Sarkar (2023) investigated an emissions-oriented SCM model where the price of the dollar exchange rate dependent on the used energy price in that SCM. Sarkar and Guchhait (2023) developed a green SCM with carbon tax and carbon cap-and-trade (CAPT) regulations, where information asymmetry is reduced by the technology support through a radio frequency identification (RFID). In the proposed work, due to production and inventory, some carbon is emitted, which is regulated by considering a cost reduction technology. Finally, the aggregate profit of the SCM model under the barter exchange policy is investigated in the presence of all such indicators: delayed period, order size, production rate, and several investments.

1.1. Research gaps

Several researchers have considered trade-credit policies in various SCMs under stochastic demand to make SCMs more flexible, increase sales and discover new customers. Some works prove through statistical data that the barter market is an effective platform to exchange overstock inventory and meet the desired profit. For stochastic demand, flexible production is required because flexible production can handle overstock and understock situations. Several SCMs were developed considering flexible production and quality improvement policies. Some gaps have been noticed in the previous literature, and they have been itemized below:

1. Several researchers introduced the concept of trade-credit and default risk in their SCM models (Kaur, 2019; Wang et al., 2021). However, a major gap in the literature is an SCM including credit period, default risk, and barter exchange policy.
2. Several SCMs were discussed under flexible production rates and stochastic demand (Tayyab et al., 2020; Schlosser and Chenavaz, 2023). However, an SCM with stochastic demand, trade credit, flexible production, and barter exchange policy is still a research gap.
3. Several researchers controlled carbon emission in their SCMs under trade-credit and variable demand (Mansouri et al., 2012). However, a significant gap in the literature is an SCM with stochastic demand, trade-credit, variable production, barter exchange policy, and carbon emission reduction.

1.2. Contribution

The proposed model makes some innovative contributions to address the gaps in previous research and develop knowledge in this field.

- The proposed model integrates trade-credit and barter exchange policies into an SCM to receive more orders and handle overstock products. The model focuses on the flexible production to meet uncertain demand.
- The proposed model develops an SCM with credit period dependent orders and controllable carbon discharges.
- Many SCMs consider several investments: discrete investments to reduce setup costs and continuous investments to enhance product quality. However, a sustainable SCM with a discrete investment for minimizing setup costs and two ongoing investments for reducing order costs and enhancing product quality significantly contribute to the literature.

This study has made these contributions to optimize the system's profitability and maintain environmental sustainability.

1.3. Orientation of the paper

The remainder of this paper is organized as follows:
Section 2 describes the literature review part briefly. Section 3 discusses the assumptions, problem definition, and notation used in this study. Section 4 formulates the mathematical model and solves it analytically to find the global optimum. Numerical examples with sensitivity analyses of different input parameters are discussed in the 5th and 6th Sections. Section 8 establishes managerial insights and implications. Finally, Section 9 lists the conclusions of this study and discusses the future scope of research.

2. Literature review

A section-wise brief review about the barter exchange policy within the SCM is described here.

2.1. Supply chain management

In operations research and applied economics, stochastic SCM plays an important role in determining the optimal inventory levels and reaching the maximum profit. Numerous authors have widely provided a variety of extensions to the SCM models under stochastic demand. A single-stage two-tier SCM with price-based stochastic demand was designed by Arcelus et al. (2008). In their model, a manufacturer offered a buyback agreement to a retailer for the remaining products at the last of the sales season, but they did not consider trade-credit and flexible production. Their model discussed the secondary market's advantage in maximizing the system's optimal profitability. Wu (2013) explored two competing SCM models with stochastic demand and buyback/non-buyback policies, but they did not focus on trade credit, flexible production, and marginal reduction technology. Zhang et al. (2014) determined the optimal order size in a single-stage SCM with fuzzy random demand in the centralized-decentralized cases. Two-stage buyback policy was introduced in their model to maximize the entire system's total profit. Sarkar et al. (2022a) introduced a circular economy approach to reduce waste in a two-level SCM intending to optimize the retailer's overall profit. In their work, they discussed deterministic demand but they did not consider stochastic demand, trade-credit policy, and flexible production. Matsumoto et al. (2024) examined a pass-through behavior of retailers within an SCM. Manufacturers and retailers payed attention to the wholesale price of products. They paid special attention to the conditions for profit maximization through Nash game but did not adopt barter exchange and marginal reduction technologies. To reduce the setup cost, Dey et al. (2021b) invested in a two-layer SCM under stochastic demand. They used two different safety factors to ignore shortages and applied multiple probability distributions to address their model. A three-layer SCM under stochastic demand and return was demonstrated by Ullah et al. (2021). They explored economic and environmental advantages of hybrid manufacturing-remanufacturing with transport packaging options but did not consider flexible production and barter exchange policy. Ullah and Sarkar (2020) introduced RFID technology in an SCM with stochastic demand to deal with unreliable retailers and traced the movement of each product on a real-time basis. Their model considered the collection used products using the proposed RFID and reused RFID tags to reduce the RFID system cost. A three-tier SCM with uncertain demand, shortage, and transportation costs was designed by Moayedi and Sadeghian (2023). In their model, they paid special attention to energy consumption (EC) reduction and carbon emission control (CEC). However, they did not consider flexible production and barter exchange policy. All these works were the same in the sense that they determined the optimal order volume. Although there are many extensions of SCM in the literature, best to the authors' knowledge, no study has discussed the case where retailers sell excess products on barter platforms. This is an important research gap that is discussed in the coming section.

2.2. Barter exchange policy under supply chain management

A barter exchange is a cashless exchange platform where any two parties can trade their excess goods or services directly based on price and equivalent estimated goods without any intermediation of money. Simply put, barter trading involves directly selling goods without using cash. The exchange is mutual, and the transaction is negotiated such that each party receives what it wants in exchange for what it offers. Barter trading platforms are used in the medical field for kidney exchange, in education for course exchange, in the power industry for energy exchange, secondary car exchange, house exchange, airlines exchange, and many more. The barter economy mainly focuses on people's basic needs, not only on financial growth. Starr (1989) described the framework of exchange in barter and monetary economy. Abraham et al. (2007) discussed the clearing algorithm to enable nationwide kidney exchange on the barter platform. Anderson et al. (2014) introduced a dynamic structure of a barter platform where in every period, one broker arrives with a single type of product to exchange for the products he needs, but they did not focus on trade credit policy. An SCM with a barter option was designed by Hua et al. (2020). In their model, the retailer faced stochastic demand and sold its excess products and purchased other necessary products from the market, but they did not consider flexible production and quality improvement policy. Bieniek (2021) inaugurated a barter exchange policy in a price-setting SCM with additive uncertainty in demand, but they did not consider trade-credit, flexible production, and marginal reduction technology. Zhang et al. (2022b) explored the effects of barter exchange policy on a two-tier SCM involving pull contracts. The manufacturer took the inventory risk and utilized the Stackelberg game strategy to solve their model. Still, they did not consider trade-credit, quality improvement, and flexible production. According to the author's cognition, barter exchange policies were introduced in many SCM models, including stochastic demand, to avoid obsolescence and deterioration of excess products. However, approving a credit payment policy from the manufacturer's aspect to increase sales, reduce holding costs, and meet the unpredictable demand is a research gap that has been studied in the next section.

2.3. Trade-credit for supply chain management

Trade-credit is a business agreement where a consumer can purchase a product with 0% financing and is contracted to pay the full amount within the due date. Trade-credit allows any SCM to be more flexible, adapt to market demand, and discover new customers such that SCM can have an uninterrupted supply of products despite its financial instability. Several researchers extensively studied different models under trade-credit. An optimal ordering policy with trade-credit was developed by Tiwari et al. (2019), but barter exchange policies was not contemplated in their model. They found solutions based on different trade-credit period with respect to the cycle time. Priyan and Uthayakumar (2014) demonstrated a vendor-buyer SCM under backorder price and transportation cost discount with investment to reduce ordering costs of the system. In their work, a trade-credit policy was provided by the vendor to the buyer to promote their business in a competitive market but flexible production and quality improvement policy were not considered. An integrated SCM with three-tier trade-credit and credit amount & credit period-based demand was designated by Pramanik et al. (2017). Their model was formulated in a harsh-crisp-fuzzy environment, and the particle swarm optimization (PSO) algorithm was used to find the solution of the model but they did not consider credit period-based order and flexible production. Sarkar et al. (2020) designed an SCM model with advertisement-driven market scenario. However, they did not discuss the concept of default risk, flexible production, and credit period-based order in their model. A three-layer SCM under two-tier trade-credit and credit-period-based demand was designed by Kishore et al. (2022). Still, they did not consider flexible production and quality improvement policy. In general, the manufacturer provides a delayed payment option without any inquiries about the actual credit status of the retailer. Due to this miscommunication, the retailer may fail to pay the dues within the delay period and in that case, interest is charged for non-payment in full within the delay period. Wu et al. (2016) examined a three-layer SCM for the deteriorating products with trade-credit and DCF policy. In their work, the retailer acquired a full trade-credit from the supplier and offered a partial trade-credit to credit-risk consumers but they did not appraise credit period-based order and quality improvement policy. A two-layer SCM with two-tier trade credit and default risk was elaborated by Kaur (2019). The demand for their model was uncertain, and the shortage was allowed. Huang et al. (2021) investigated a two-layer SCM involving one manufacturer and retailer under credit sales and stochastic demand. The retailer exchanged the unsold goods for needed subsidiary products on the barter market in their model. They demonstrated that retailers can increase profits through bartering when faced with highly uncertain demand. Gao et al. (2023) discussed a case study of information asymmetry in the trade-credit process. They explained how a week information sharing affects the bargaining policy for trade-credit among two SCM players. Still, they did not consider flexible manufacturing and investments to reduce ordering and setup costs and improve information quality. Several researchers investigated different SCMs under trade-credit, default risk, and barter exchange policy. However, an SCM under trade-credit, default risk, barter exchange policy, and sustainability is not yet considered by any existing literature. Thus, this work has made an effort to fill the research gap.

Table 1
Author(s) comparison table.

Author(s)	Demand pattern	Trade credit	Production type	Shortage	Strategy	Investments
Mishra et al. (2020)	Fixed	NC	Fixed	Allowed	Green technology	For CEC
Hua et al. (2020)	Stochastic	NC	NC	Allowed	Barter exchange	NC
Marculetiu et al. (2023)	Review	NC	Fixed	NA	Pollution	For risk
Dey et al. (2021a)	Variable	NC	Flexible	Allowed	Smart production	For SCR, PPR
Huang et al. (2021)	Stochastic	Considered	Fixed	NA	Barter exchange	NC
Sepehri et al. (2021)	Variable	NC	Fixed	NA	Preservation, CEC	For QI, CEC
Ullah et al. (2021)	Stochastic	NC	Fixed	NA	Waste control	NC
Chen et al. (2024)	Uncertain	NC	NC	Fixed	Back-up sourcing	NC
Sarkar et al. (2022b)	Stochastic	NC	Fixed	NA	REM	NC
Al-e-hashem et al. (2013)	Fixed	NC	Flexible	Allowed	Discount, flexible LT	NC
Kar et al. (2023)	Variable	NC	Flexible	NA	O2O retailing, carbon tax, CAPT policy	For GT and advertising
Gao et al. (2023)	Empirical	Considered	NC	NC	Bargaining policy	For credit restoration
Kugele and Sarkar (2023)	Fixed	NA	Flexible	NA	Smart production, ATN,CE and FC control, RE	For SCR, FC control
Zhang and Antonopoulos (2013)	Fixed	NA	Fixed	NA	Barter game	NA
This model	Stochastic	Considered	Flexible	Allowed	Barter exchange, CEC	For QI, SCR, OCR

QI - quality improvement, RFID - radio frequency identification, SCR - setup cost reduction, OCR - ordering cost reduction, PPR - production process reliability, REM - remanufacturing, BFP - biofuel production, ATN - autonomation, CEC - carbon emission control, FC - fuel consumption, GT - green technology, RE - renewable energy, LT - lead time, NC - not considered, and NA - not applicable.

2.4. Sustainable supply chain management

The concept of sustainability integrates the ethical and environmental responsibilities of SCM in a competitive business market. It not only focuses on environmental protection but also contemplates on maximum profitability of an SCM. Several researchers developed various sustainable SCMs in order to reduce carbon discharges, manage waste, and avoid environmental disruption. Yadav et al. (2021) controlled carbon emissions in a sustainable SCM to make the environment cleaner. They used waste and emissions reduction policy with a cross-price elasticity. Mansouri et al. (2012) exhibited a green inventory model of non-perishable items. They discussed how green inventory helps SCM to maximize the gross profit under several trade-credit policies while reducing carbon emissions for a cleaner environment. To control pollution and minimize the total cost of the system, Marculetiu et al. (2023) reviewed factors and their pressure on a sustainable SCM. Moreover, by investing a certain amount, they proved that relation between SCM players could be improved for achieving sustainability. Sepehri et al. (2021) made some investments in their production to alleviate carbon emissions and improve product quality. In their model, a fraction of manufactured products were imperfect, and they applied preservation technology to control the degradation of perishable items of poor quality. Sarkar et al. (2021) focused on a sustainable SCM of fixed lifetime products under the waste reduction concept. They improved product quality and reduced production setup costs by considering some investments. An algebraic method was applied to solve their model and found the optimum global result. A biodiesel supply chain design was investigated by Habib et al. (2021), where they used waste products as raw material. They established a sustainable supply chain by reducing and reusing waste. Kar et al. (2023) applied the CAPT strategy in a flexible production model for single-type substitutable products with several investments for green technology and advertising purposes. In their model, the demand was dependent on an online-offline selling price and an online advertisement of the product. Several researchers discussed different SCMs under controllable carbon discharges, trade-credit, stochastic demand, and barter exchange policy. However, an SCM with stochastic demand, trade-credit, controllable carbon discharges, and flexible production is still a significant research gap that has been filled in the next section.

2.5. Flexible production for sustainable supply chain management

One of the vital decisions taken by the production manager in the supply chain is to make the flexible manufacturing system smarter and meet the uncertain demand. Flexible production helps the manufacturing company handle both overstock and understock situations and ensure product availability to the customer. Numerous researchers developed different SCMs under flexible production. Yang et al. (2023) studied a flexible jobshop production model with digital twin. They established that flexible production is more efficient for job scheduling but they did not consider environmental perspective for sustainability. Dey et al. (2021a) designed a smart SCM under advertise-based demand, flexible production, and variable lead time and variance. They used a flexible production and made some investments in their model to minimize setup expenditure and enhance the manufacturing process reliability. A flexible production with cloud computing was studied by Ma et al. (2023). In their work, they promoted environmental, social, and governance (ESG) for an energy-saving flexible production, but they did not consider payment policy and barter exchange policies. Chen et al. (2024) managed uncertain demand in an SCM considering shortage and found back-up sourcing for reducing risk. They did robust optimization to test the stability of the risk. A customer-centric two-stage SCM were studied by Garai and Sarkar (2022) with the consideration of carbon emissions from the system. They considered the waste of first-stage SCM as raw materials for second-stage SCM. Sarkar and Bhuniya (2022) exhibited a manufacturing-remanufacturing SCM under flexible production and stochastic demand. Their SCM was developed through some green investment and a waste control venture. A flexible production system with lead time and transportation was studied by Al-e-hashem et al. (2013). They considered quantity discount policy within a period but periodically within that period. They established relationship between lead time, transportation of products, and carbon emissions. Based on previous research, it is clear that numerous authors considered flexible production in their SCMs to meet the unpredictable demand and reassure customers about product availability. However, the existing literature still did not consider flexible production in SCM, including stochastic demand, trade-credit, and barter exchange policy. An attempt has been made in the proposed model to fill this research gap.

This proposed work designs an SCM considering stochastic demand and barter exchange policy. This model calculates and optimizes the total profit concerning the credit period, product quality improvement, production rate, and different investments. This SCM pays a special attention to carbon emissions reduction for environmental protection. In addition, this model considers the possibility of default risk and shortage occurrence due to trade-credit and uncertain demand. Best to the authors' knowledge, previous researchers did not appraise barter exchange policy, credit

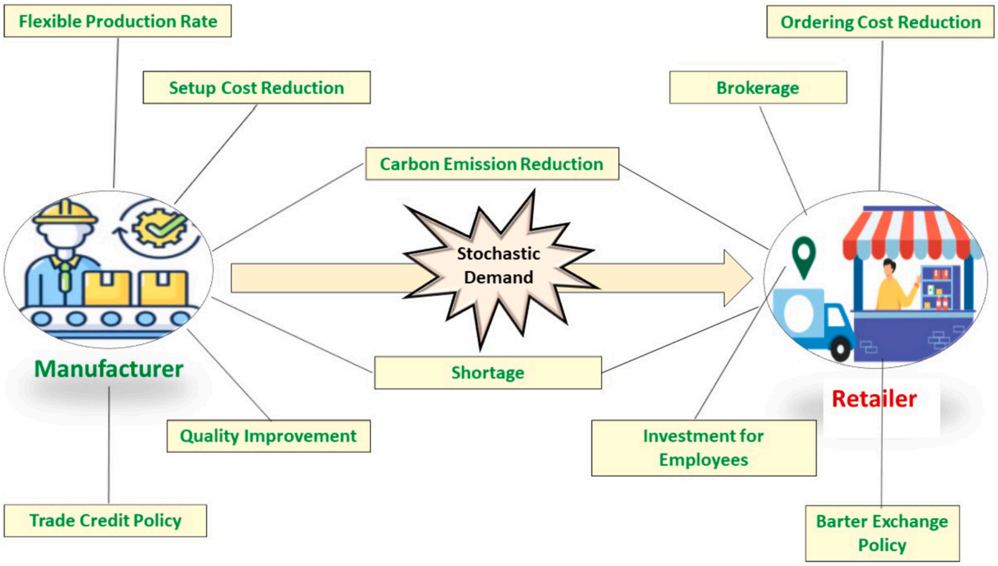

Fig. 2. Integrated SCM under credit sales and barter exchange.

period, default risk, flexible production, barter exchange policy, carbon emission reduction, quality improvement policy, and several investments for ordering and setup costs reduction in the same frame. However, these issues are very important for any SCM to deal with uncertain demand, increase sales, maximize profits, maintain environmental sustainability, and improve product quality. Thus, the model is more useful, profitable, and acceptable in every aspect than the others.

Some previous research works done in this field are given in Table 1.

3. Problem definition, notation, and assumptions

This section discusses problem definition along with notation and assumptions for the model. The problem definition is mentioned in detail first, followed by a brief discussion on the notation and assumptions.

3.1. Problem definition

A major problem in modern civilization is waste, which comes not only from consumable products but also from overstocked inventory. One of the main reasons for overstocking inventory is the uncertain demand situation. Managing overstock products and dealing with uncertain demand situation becomes a major challenge for the industry. Keeping these issues in mind, this study attempts to maximize profits by solving these problems. Fig. 2 illustrates the problem definition of the proposed model through a diagram. The main objective behind the study is to manage overstock products and handle uncertain demand situations. This model discusses an SCM consisting of a single manufacturer and retailer with a credit granting strategy from the manufacturer's perspective and a barter exchange strategy from the retailer's perspective. Here, the manufacturer first chooses a credit payment strategy consisting of two inputs: an initial payment to guarantee the product and then a delayed payment to encourage the retailer to sign the contract and place the maximum order. That is, order volume depends on credit period in this study. The manufacturer uses another effort like flexible production to meet the uncertain demand. Again, in practice, it is a very common scenario that the retailer needs to purchase some products from any market for employees, such as for office purposes or other products that are required. Here, the retailer prefers a barter exchange platform to exchange his surplus products through a broker for the products he needs at almost full retail price. Various investments are focused on this task to reduce setup, ordering costs and improve product quality. In addition, marginal reduction technology is introduced in this work to limit carbon emissions.

3.2. Notation

All symbols of this model are described in Table 2.

Table 2
Notation.

	Decision variables
m_d	credit period granted by the manufacturer to retailer (year)
q	order size (units)
K	variable investment to diminish the setup cost of manufacturer ($)
P_m	production rate per unit time of manufacturer (units/unit time)
q_i	improvement of quality
A	variable investment to diminish the ordering expenditure of retailer ($)

	Random variables
x	stochastic demand
	Parameters
	Notation of manufacturer
I_p	Retailer's initial payment to the manufacturer for product assurance ($)
B_q	basic order quantity (units)
ω	credit sales sensitivity coefficient
C_{ome}	manufacturer's ordering expenditure per order ($/order)
C_{sme}	manufacturer's setup expenditure ($/setup)
U_b	ratio of setup expenditure and setup time
$S(K)$	discrete function of investment assumed as an expression of the variable investment (K) as $S(K)=U_b e^{-\alpha_1 K}+K$
α_1	shape parameter for the investment to diminish the setup expenditure of the manufacturer
α_{cm}	manufacturer's annual compound interest rate on opportunity cost
α_{3q}	coefficient of cost for product quality investment (> 1)
θ_{1p}	shape parameter of investment to improve the quality of product
g_l	goodwill lost expense of the manufacturer ($/unit)
m_w	manufacturer's unit wholesale price ($/unit)
C_{hmne}	manufacturer's unit holding expenditure ($/unit/unit time)
C_{1m}	manufacturer's unit carbon cost in the carbon trading market ($/unit)
I_{cmp}	initial carbon emission of each product (units)
I_{gmp}	emissions target set by the manufacturer (units)
b_m	per unit marginal reduction cost ($/unit)
$U(P_m)$	production cost function for each unit ($/unit)
B^P_{rme}	unit raw material expense for manufacturing ($/unit)
L_{im}	development cost of the production system ($/unit)
α_{im}	unit die/tool cost ($/unit)
ρ	shape parameter for tool/die cost
s_{1m}	shortage penalty cost per unit ($/unit)
	Notation of retailer
l	mean of demand
ϑ	standard deviation of demand
A_{0r}	initial ordering expenditure of the retailer ($/order)
L_b	ratio of ordering cost and ordering time
$S(A)$	discrete function of investment assumed as a function of the variable (decision) investment A as $S(A)=L_b log(A_{0r}/A)+A$
C_{hrbe}	per unit holding expense per unit time in classical robust inventory model and robust inventory model with barter exchange ($/unit/unit time)
C_{2r}	unit carbon cost of the retailer in carbon trading market ($/unit)
I_{cre}	initial carbon emission of each product (units)
I_{gre}	emissions target set by the retailer (units)
a_r	per unit marginal reduction cost ($/unit)
θ_l	coefficient of default risk of the retailer
p_r	per unit selling price ($/unit)
s_{2r}	per unit shortage penalty cost ($/unit)
q_{0r}	price of the items the retailer needs on barter platform is equal to the price of q_{0r} units of the product the retailer sells ($/unit)
r	retailer bares r percent of the retail price to broker on the barter platform as a commission for every product
	Other notation
R_p^{TSB}	retailer's profit for the SCM with barter exchange ($)
M_p^{TSB}	manufacturer's profit for the SCM with barter exchange ($)
J_p^{MRCB}	joint profit (retailer and manufacturer) for the SCM model with barter exchange ($)
$x+$	maximum value of x and 0
$E(.)$	mathematical expectation

3.3. Assumptions

The assumptions related to the model are provided below.

1. This study illustrates an SCM under a trade-credit policy granted by the manufacturer (Huang et al., 2021) and a barter exchange policy considered by the retailer (Hua et al., 2020). The order volume q of the retailer is assumed as $q = B_q e^{\omega m}$, where B_q is the basic order volume, and ω is the credit sales sensitivity coefficient (Li et al., 2018).
2. The demand pattern is stochastic and due to it, the shortage arises (Liu et al., 2023) in this study. This model makes several investments to control the manufacturer's setup expenditure and the retailer's ordering expenditure. Furthermore, the product quality is improved in this study by considering a certain investment (Sepehri et al., 2021).
3. The production rate in this study is considered as variable (Yadav et al., 2021) and the unit production cost is $U(P_m) = B^P_{rme} + L_{im}/P_m + \alpha_{im} P^{\rho}_m$ where, B^P_{rme} is the unit raw material price at manufacturer for manufacturing, L_{im} is the development cost for each cycle of the production process and $\alpha_{im} P^{\rho}_m$ is tool or die cost. Increasing production rate, tool/die cost, and decreasing development cost are shown in the expression of production cost per unit. In this SCM, each player is responsible for the marginal reduction of carbon emissions. The retailer and manufacturer both have separate operational expenditures for the reduction task. These are quadratic in reduction amount $(I_{cmp} - I_{gmp}), (I_{cre} - I_{gre})$ (Ghosh et al., 2020).
4. It is assumed that the price of the items that the retailer needs on the exchange platform is equal to the selling price of q_{0r} units of the retailer (Hua et al., 2020).

5. The continuous function of investment for ordering cost reduction of the retailer assumed as a function of the variable investment A as $S(A) = L_b log(A_{0r}/A) + A$, where L_b is the ratio of ordering cost and ordering time and A_{0r} is the retailer's initial ordering cost. The function $S(A)$ is convex and has a minimum value at L_b.
6. The retailer trades his products on barter platform at the retail price, and for this purpose, he gives a commission of rp_r for each product to the broker on the forum. Therefore, the retailer's selling price of each product on the barter platform is $(1-r)p_r$. It is adopted that $(1-r)p_r > m_w$ suggests that the retailer can be more beneficial for choosing the barter platform.

4. Model formulation

This section is formulated and solved a sustainable SCM under trade-credit and barter exchange policy.

In this SCM, the manufacturer grants a credit payment facility for the retailer, and the retailer then signs the contract and places an order for q products. Here, the retailer initially pays some price I_p as an assurance cost and negotiates to pay the outstanding amount for the m_d credit period. Next, the manufacturer constructs an infrastructure to produce q items considering some investment to minimize setup expenditure and enhance product quality. In this model, the retailer's actual credit status is unknown to the manufacturer but the manufacturer offers the delayed payment facility to the retailer. Hence, there is potential for credit default risk on the retailer's side. In this situation, the manufacturer takes a compound interest from the retailer for the default period. Moreover, the production rate is presumed to be variable, and the manufacturer pays some abatement costs to limit the number of carbon emissions caused by the production. Then, the manufacturer dispatches the finished goods to the retailer. The manufacturer pays the shortage penalty cost if the demand exceeds the order size. The manufacturer holds the excess products if the product quantity exceeds the demand.

After signing the credit payment agreement, the retailer orders for q items from the manufacturer with a lower ordering cost by adopting a continuous investment. Furthermore, the retailer pays the finished product's purchasing price, carbon emission expenditure linked to inventory, and carbon emission reduction cost to control the carbon emissions. The demand x is stochastic. If the demand x exceeds the order size q, the retailer faces shortages, and if $x < q$, the retailer selects a barter exchange platform to sell unsold products. However, on the barter exchange platform, the retailer buys some goods for his firm's employees whose value is the price of q_{0r} unit products. After that, if there is still some unsold products, the retailer stores them at a holding cost of C_{hrbe} per unit.

4.1. *Manufacturer's model*

The manufacturer grants a credit period for the retailer to get large orders and considers flexible production to meet unpredictable demand. Primary expenditures like ordering and setup are included with the manufacturer's additional cost. All costs associated with the manufacturer are described below:

4.1.1. *Ordering cost (OMB)*

An important cost for purchasing raw materials for production is the ordering cost. In this work, the manufacturer buys raw material at a fixed order price

$$OMB = C_{ome}. \tag{1}$$

4.1.2. *Setup cost (SMB)*

Setup expenditure is a significant cost considered at the beginning of any business to set up the infrastructure for production and run the process smoothly. In this work, the manufacturer sets an infrastructure at the beginning of the production process at a certain setup cost

$$SMB = C_{sme}. \tag{2}$$

4.1.3. *Investment for setup expenditure reduction (ISR)*

The setup expenditure becomes significantly higher in a manufacturing process, which increases the total system cost. In this study, the manufacturer considers a discrete investment to reduce the setup expenditure, and the discrete function of investment to reduce the setup costs is revealed as follows:

$$ISR = U_b e^{-\alpha_1 K} + K. \tag{3}$$

4.1.4. *Production cost (PMB)*

In reality, the demand for any product in a competitive market is not always the same. This study considers a flexible production to meet uncertain demands. In such cases, the production cost per unit is contemplated as an expression of production rate, development cost, tool/die cost, and raw material cost. Development costs are usually invested in adopting new technologies to improve production. Again, to keep the production process smooth and avoid any machinery problems, it is very efficient to check the machinery regularly. The cost for this purpose is called tool/die cost. The manufacturer considers flexible production to produce q products in this work. The manufacturer considers flexible production to have q products in this work. Thus, using the Assumption 3, the production cost of q products is expressed as follows:

$$PMB = q\left\{B^P_{rme} + \frac{L_{im}}{P_m} + \alpha_{im} P_m^{\rho}\right\}. \tag{4}$$

4.1.5. *Investment for quality improvement (IQIB)*

Again, the demand for any product fluctuates based on the product's quality. Consequently, the manufacturer invests in improving product quality, which is revealed as follows:

$$IQIB = \alpha_{3q}\frac{q_i^{\theta_{1p}}}{2}, \alpha_{3q} > 1. \tag{5}$$

4.1.6. Goodwill loss cost (GLB)

The quality is not hundred percent pure for all merchandise. Some impurities $(1-q_i)$ are involved in the quality, and the manufacturer's goodwill is affected. This results in a goodwill loss cost which is revealed as follows:

$$GLB = g_l(1-q_i). \tag{6}$$

4.1.7. Carbon discharge cost (CDB)

During production, huge amounts of carbon are discharged, which affects the environment and the ecosystem. In this work, the manufacturer fixes a carbon emission goal of I_{gmp} units per product to limit the amount of carbon discharges during the production of q items and bares the carbon cost of C_{1m} per unit. Thus, the manufacturer's carbon discharges cost is revealed as follows:

$$CDB = qC_{1m}I_{gmp}. \tag{7}$$

4.1.8. Marginal reduction cost (MRB)

This work focuses on the marginal mitigation of carbon discharges for environmental protection with a specific reduction cost of b_m per unit. Again, the initial carbon discharges of each product during production are considered as I_{cmp} units. Therefore, by Assumption 3, the marginal reduction cost of q items paid by the manufacturer to limit the carbon discharges is expressed as follows:

$$MRB = qb_m(I_{cmp}-I_{gmp})^2. \tag{8}$$

4.1.9. Shortage cost (SCB)

If the manufacturer has less stock than the product demand, it faces shortages. In this situation, the manufacturer's image is lost, and it has to pay the shortage penalty cost. Here, when $x \geq q$, the manufacturer faces a shortage, and the shortage penalty cost conferred by the manufacturer is expressed as

$$SCB = s_{1m}(x-q)^+. \tag{9}$$

4.1.10. Holding cost (HCB)

If the manufacturer has excess stock than the product demand, it faces an overstock situation. Then to save the overstock inventory, it is very efficient to store excess products with some holding costs. Here, when $x < q$, the manufacturer holds extra products with a certain holding cost of C_{hmne} per unit. Therefore, the total holding expenditure conferred by the manufacturer for stocking the overstock products is expressed as follows:

$$HCB = C_{hmne}(q-x)^+. \tag{10}$$

4.1.11. Revenue (RVB)

Revenue refers to the gross income of the manufacturer from which all expenses are excluded to obtain the desired profit. In this work, the manufacturer receives the initial payment I_p and the late payment $m_w q e^{-(\theta_l+\alpha_{cm})m_d}$, when $x \geq q$ and $m_w x e^{-(\theta_l+\alpha_{cm})m_d}$, when $x < q$ from the retailer. Thus, the revenue of the manufacturer under two conditions is

$$RVB = \begin{cases} I_p + m_w q e^{-(\theta_l+\alpha_{cm})m_d}, & \text{if } x \geq q \\ I_p + m_w x e^{-(\theta_l+\alpha_{cm})m_d}, & \text{if } x < q. \end{cases} \tag{11}$$

4.1.12. Manufacturer's profit

The manufacturer's aggregate profit can be gained by deducting all expenses from the revenue; therefore, the manufacturer's aggregate profit is

$$\begin{aligned} &M_p^{TSB}\left(m_d, q, K, P_m, q_i\right) \\ &= \begin{cases} I_p + m_w q e^{-(\theta_l+\alpha_{cm})m_d} - [C_{ome} + C_{sme} + U_b e^{-\alpha_1 K} + K + \alpha_{3q}\dfrac{q_i^{\theta_{1p}}}{2} + g_l(1-q_i) + qC_{1m}I_{gmp} + qb_m(I_{cmp}-I_{gmp})^2 \\ +q\left\{B_{rme}^P + \frac{L_{im}}{P_m} + \alpha_{im}P_m^\rho\right\} + s_{1m}(x-q)^+], & \text{if } x \geq q \\ I_p + m_w x e^{-(\theta_l+\alpha_{cm})m_d} - C_{hmne}(q-x)^+ - [C_{ome} + C_{sme} + U_b e^{-\alpha_1 K} + K + \alpha_{3q}\dfrac{q_i^{\theta_{1p}}}{2} + g_l(1-q_i) + qC_{1m}I_{gmp} + qb_m(I_{cmp}-I_{gmp})^2 \\ +q\left\{B_{rme}^P + \frac{L_{im}}{P_m} + \alpha_{im}P_m^\rho\right\}], & \text{if } x < q. \end{cases} \end{aligned} \tag{12}$$

Thus, the manufacturer's expected profit is

$$\begin{aligned} &E[M_p^{TSB}\left(m_d, q, K, P_m, q_i\right)] = \\ &I_p + m_w e^{-(\theta_l+\alpha_{cm})m_d} E[min(x,q)] - C_{ome} - C_{sme} - U_b e^{-\alpha_1 K} - K - \alpha_{3q}\frac{q_i^{\theta_{1p}}}{2} - g_l(1-q_i) - q\left\{C_{1m}I_{gmp} + b_m(I_{cmp}-I_{gmp})^2 + B_{rme}^P + \frac{L_{im}}{P_m} + \alpha_{im}P_m^\rho\right\} \\ &-s_{1m}E(x-q)^+ - C_{hmne}E(q-x)^+. \end{aligned} \tag{13}$$

4.2. Retailer's model

Another important player of the SCM is the retailer. The retailer signs the delayed payment agreement and orders for q finished products to the manufacturer. Since the demand pattern is stochastic, the retailer does not hold the unsold products when the retailer's inventory exceeds the customer's demand but chooses a barter platform to sell or exchange excess products. The retailer buys some products for his firm's employees whose value is the price of q_{0r} unit products he sells, i.e., $p_r q_{0r}$, where p_r is the retailer's per unit selling price. The most important factor in this model is that the retailer offers a commission of rp_r per product to the broker in the barter platform to barter excess products for other products he needs. Therefore, the value per unit product sold by the retailer in the barter platform is $(1-r)p_r$. The retailer pays some important costs such as initial ordering cost, investment to reduce ordering cost, purchasing price of finished products, shortage cost, carbon emission expenditure due to inventory, and carbon emission reduction cost. All expenses related to SCM's retailer are discussed below.

4.2.1. Ordering cost (OB)

Ordering expenditure is the amount that is incurred on placing and processing an order. In this study, the retailer's initial ordering cost is assumed as A_{0r} and the retailer makes a continuous investment to reduce the ordering expenditure. The continuous function of investment to lessen the ordering cost is expressed as follows:

$$OB = L_b log\left(\frac{A_{0r}}{A}\right) + A. \tag{14}$$

4.2.2. Carbon emission cost (CEB)

Due to retailers' inventory, some amount of carbon is discharged, which is harmful to the environment. Here, The retailer fixes a carbon emission goal of I_{gre} units per product to limit carbon discharges due to inventory of q products and bares the carbon cost of C_{2r} per unit. Thus, the carbon discharge cost paid by the retailer is revealed as follows:

$$CER = qC_{2r}I_{gre}. \tag{15}$$

4.2.3. Carbon emission reduction cost (CERB)

This study pays special attention to the marginal reduction of carbon discharges, with a specific reduction cost of a_r per unit. Again, due to inventory, the initial carbon discharges of each product are assumed as I_{cre} units. Therefore, by Assumption of 4, the marginal reduction cost of q items paid by the retailer to limit the number of carbon discharges is expressed as follows:

$$CERB = qa_r(I_{cre} - I_{gre})^2. \tag{16}$$

4.2.4. Shortage cost (SPR)

When $x \geq q$, i.e., customer demand exceeds the retailer's order size q, the retailer spends $s_{2r}(x-q)^+$ deficit as shortage cost on unsatisfied demand. Due to uncertain demand x, a shortage situation occurs when $x \geq q$, i.e., customer demand x exceeds the retailer's order size q. Here, the per unit shortage penalty cost paid by the retailer is presumed as s_{2r}. Thus, the total shortage cost conferred by the retailer is revealed as follows:

$$SPR = s_{2r}(x-q)^+. \tag{17}$$

4.2.5. Purchasing price of necessary products for employees or office purposes (PPR)

It is common for a retailer to purchase certain products for their employees or office purposes. When there is an option of going barter platform, the retailer accepts the opportunity, and when there is no such option, the retailer purchases products from any market. In this study, If $x \geq q$, the retailer faces a shortage, and in this case, the retailer does not go to the barter market and purchases the necessary products from any market. Again, if $x < q$, the retailer faces an overstock situation. In this case, the retailer chooses the barter platform to exchange the unsold products for the products the retailer needs. Now, the purchasing price under different circumstances is discussed as follows:

Case 1 $x \geq q$ When $x \geq q$, the retailer does not go to the exchange platform and buys the required product from the market whose purchase price is equal to the retailer's selling price of q_{0r} products. In this case, the purchasing price of necessary products is expressed as follows:

$$PPR = p_r q_{0r}. \tag{18}$$

Case 2 $x < q \leq (x + q_{0r})$ When $x < q \leq (x + q_{0r})$, i.e., retailer's order size q exceeds demand x, $(q-x)^+$ unit products remain unsold. The retailer then exchanges $(q-x)^+$ unit products for the products he wants on the barter exchange platform and purchases products from any market whose price is equal to the retailer's selling price of $(q_{0r} - (q-x))^+$, i.e., $(x - (q - q_{0r}))^+$ products. In this case, the retailer's purchasing price is expressed as follows:

$$PPR = p_r(x - (q - q_{0r}))^+. \tag{19}$$

[For reference, see Hua et al. (2020).]

4.2.6. Holding cost (HRB)

When $q > x + q_{0r}$, the retailer exchanges his unsold items in the barter market for all the products it needs and stores the rest $(q - x - q_{0r})^+$ i.e., $((q - q_{0r}) - x)^+$ products at a holding cost of C_{hrbe} per unit. Thus, the retailer's total holding expenditure is expressed as follows:

$$HRB = C_{hrbe}((q - q_{0r}) - x)^+. \tag{20}$$

4.2.7. Commission to the broker (CBB)

Case 1 $x < q \leq (x + q_{0r})$: When $x < q \leq (x + q_{0r})$ i.e., the retailer's order size q exceeds the customer's demand x, there is $(q-x)^+$ units of excess product. The retailer then trades these $(q-x)^+$ unit products on the barter platform, where the commission for each product paid by the retailer is treated as r% of per-unit retail price p_r. Thus, the total commission bestowed by the retailer, in this case, is expressed as follows:

$$CBB = rp_r(q-x)^+. \tag{21}$$

Case 2 *$q > x + q_{0r}$:* When $q > x + q_{0r}$, the retailer barters his products at a commission of rp_r per product in the barter platform to barter excess products for other products it needs. Thus, the total commission bestowed by the retailer to the broker, in this case, is expressed as follows:

$$CBB = rp_r q_{0r}. \tag{22}$$

4.2.8. Revenue (RVR)

Due to the stochastic demand x, two possibilities arise: $x \geq q$ and $x < q$. Here, the retailer's selling price per unit is considered to be p_r. Then the retailer's revenue under two conditions is expressed as follows:

$$RVR = \begin{cases} p_r q, & \text{if } x \geq q \\ p_r x, & \text{if } x < q. \end{cases} \tag{23}$$

4.2.9. Retailer's profit

The retailer's aggregate profit can be gained by deducting all expenses from the revenue; therefore, the retailer's aggregate profit is

$$\begin{aligned} &R_p^{TSB}(m_d, K, P_m, q_i, A) \\ &= \begin{cases} \left(p_r - m_w e^{-(\theta_l+\alpha_{cm})m_d} - C_{2r}I_{gre} - a_r(I_{cre} - I_{gre})^2\right)q - I_p - s_{2r}(x-q)^+ - L_b log\left(\frac{A_{0r}}{A}\right) - A - p_r q_{0r}, & \text{if } q \leq x \\ p_r x - rp_r(q-x)^+ - p_r(x-(q-q_{0r}))^+ - m_w x e^{-(\theta_l+\alpha_{cm})m_d} - I_p - \left(C_{2r}I_{gre} + a_r(I_{cre} - I_{gre})^2\right)q - L_b log\left(\frac{A_{0r}}{A}\right) - A, & \text{if } x < q \leq (x + q_{0r}) \\ p_r x - rp_r q_{0r} + C_{hrbe}((q-q_{0r}) - x)^+ - m_w x e^{-(\theta_l+\alpha_{cm})m_d} - I_p - \left(C_{2r}I_{gre} + a_r(I_{cre} - I_{gre})^2\right)q - L_b log\left(\frac{A_{0r}}{A}\right) - A, & \text{if } q \geq x + q_{0r}. \end{cases} \end{aligned} \tag{24}$$

After simplifying all cases, the retailer's expected profit is given by

$$\begin{aligned} &E[R_p^{TSB}(m_d, K, P_m, q_i, A)] = \\ &p_r E[min(x,q)] - I_p - m_w e^{-(\theta_l+\alpha_{cm})m_d} E[min(x,q)] - L_b log\left(\frac{A_{0r}}{A}\right) - A - qC_{2r}I_{gre} - qa_r(I_{cre} - I_{gre})^2 - s_{2r}E(x-q)^+ - C_{hrbe}E((q-q_{0r}) - x)^+ \\ &-rp_r(q-x)^+ - p_r(x-(q-q_{0r}))^+ - rp_r q_{0r} - p_r q_{0r}. \end{aligned} \tag{25}$$

4.3. Combined profit of the manufacturer and retailer

Using Equations (13), (25), the expected joint profit of the retailer and manufacturer becomes

$$\begin{aligned} &E[J_p^{MRCB}\left(m_d, K, P_m, q_i, A\right)] = \\ &p_r E[min(x,q)] - C_{ome} - C_{sme} - U_b e^{-\alpha_1 K} - K - \alpha_{3q}\frac{q_i^{\theta_{1p}}}{2} - g_l(1-q_i) - L_b log\left(\frac{A_{0r}}{A}\right) - A - q\left\{C_{1m}I_{gmp} + C_{2r}I_{gre} + b_m(I_{cmp} - I_{gmp})^2 + a_r(I_{cre} - I_{gre})^2\right. \\ &\left.+B_{rme}^P + \frac{L_{im}}{P_m} + \alpha_{im}P_m^\rho\right\} - (s_{1m} + s_{2r})E(x-q)^+ - C_{hmne}E(q-x)^+ - C_{hrbe}E((q-q_{0r}) - x)^+ - rp_r(q-x)^+ - p_r(x-(q-q_{0r}))^+ - rp_r q_{0r} - p_r q_{0r} \\ &= p_r l - C_{ome} - C_{sme} - U_b e^{-\alpha_1 K} - K - \alpha_{3q}\frac{q_i^{\theta_{1p}}}{2} - g_l(1-q_i) - L_b log\left(\frac{A_{0r}}{A}\right) - A - q\left\{C_{1m}I_{gmp} + C_{2r}I_{gre} + b_m(I_{cmp} - I_{gmp})^2 + a_r(I_{cre} - I_{gre})^2\right. \\ &\left.+B_{rme}^P + \frac{L_{im}}{P_m} + \alpha_{im}P_m^\rho\right\} - \frac{p_r + s_{1m} + s_{2r}}{2}\left[\sqrt{\vartheta^2 + (q-l)^2} - (q-l)\right] - \frac{C_{hmne}}{2}\left[\sqrt{\vartheta^2 + (q-l)^2} - (l-q)\right] \\ &-\frac{C_{hrbe}}{2}\left[\sqrt{\vartheta^2 + ((q-q_{0r}) - l)^2} - (l-(q-q_{0r}))\right] - \frac{rp_r}{2}\left[\sqrt{\vartheta^2 + (q-l)^2} - (l-q)\right] - \frac{p_r}{2}\left[\sqrt{\vartheta^2 + ((q-q_{0r}) - l)^2} - ((q-q_{0r}) - l)\right] - rp_r q_{0r} - p_r q_{0r} \end{aligned}$$

[See Sarkar and Guchhait (2023) for reference.]

$$\begin{aligned} &= p_r l - C_{ome} - C_{sme} - U_b e^{-\alpha_1 K} - K - \alpha_{3q}\frac{q_i^{\theta_{1p}}}{2} - g_l(1-q_i) - L_b log\left(\frac{A_{0r}}{A}\right) - A - B_q e^{\omega m_d}\left\{C_{1m}I_{gmp} + C_{2r}I_{gre} + b_m(I_{cmp} - I_{gmp})^2 + a_r(I_{cre} - I_{gre})^2\right. \\ &\left.+B_{rme}^P + \frac{L_{im}}{P_m} + \alpha_{im}P_m^\rho\right\} - \frac{p_r + s_{1m} + s_{2r}}{2}\left[\sqrt{\vartheta^2 + (B_q e^{\omega m_d} - l)^2} - (B_q e^{\omega m_d} - l)\right] - \frac{C_{hmne}}{2}\left[\sqrt{\vartheta^2 + (B_q e^{\omega m_d} - l)^2} - (l - B_q e^{\omega m_d})\right] \\ &-\frac{C_{hrbe}}{2}\left[\sqrt{\vartheta^2 + ((B_q e^{\omega m_d} - q_{0r}) - l)^2} - (l - (B_q e^{\omega m_d} - q_{0r}))\right] - \frac{rp_r}{2}\left[\sqrt{\vartheta^2 + (B_q e^{\omega m_d} - l)^2} - (l - B_q e^{\omega m_d})\right] - \frac{p_r}{2}\left[\sqrt{\vartheta^2 + ((B_q e^{\omega m_d} - q_{0r}) - l)^2}\right. \\ &\left.-((B_q e^{\omega m_d} - q_{0r}) - l)\right] - rp_r q_{0r} - p_r q_{0r}. \end{aligned} \tag{26}$$

4.4. Solution methodology

A classical optimization method is applied here to find the solution of the mathematical model. The system's combined expected profit is optimized for the decision variables m_d, K, P_m, A, and q_i. The Hessian matrix is calculated to vindicate the sufficient part. First, the joint profit function J_p^{MRCB} is differentiated partially concerning the decision variables. Then, equating the mentioned partial derivatives with zero, the stationary points m_d, K, P_m, A and q_i are found as follows:

$$m_d^* = \frac{1}{\omega} log \left[\frac{l + \frac{G_1(m_d, P_m)}{G_2(m_d)}}{B_q} \right] \tag{27}$$

$$K^* = \frac{1}{\alpha_1} log(U_b \alpha_1) \tag{28}$$

$$P_m^* = \left[\frac{L_{im}}{\alpha_{im}\rho} \right]^{\frac{1}{\rho+1}} \tag{29}$$

$$A^* = L_b \tag{30}$$

$$q_i^* = \left[\frac{2g_l}{\alpha_{3q}\theta_{1p}} \right]^{\frac{1}{\theta_{1p}-1}}. \tag{31}$$

[The expressions of $G_1(m_d, P_m), G_2(m_d)$ are given in Appendix B and all first order partial derivatives are given in Appendix C].

The following propositions are exploited here to validate the global optimality of the expected profit and gratify the sufficient conditions.

Proposition 1. *The principal minor of the first order of the Hessian matrix for the expected profit function is less than zero at the optimum findings of the decision variables* m_d, K, P_m, q_i, A *if* $G_3(m_d, P_m) < 0$.

Proof. For proof, see Appendix D and Appendix E.

Proposition 2. *The principal minor of second order of the Hessian matrix for the expected profit function is greater than zero at the optimum findings of the decision variables* m_d, K, P_m, q_i, A *if* $G_3(m_d, P_m)G_5(m_d, P_m) + \left\{G_8(m_d, P_m)\right\}^2 < 0$.

Proof. For proof, see Appendix D and Appendix F.

Proposition 3. *The principal minor of third order of the Hessian matrix for the expected profit function is less than zero at the optimum findings of the decision variables* m_d, K, P_m, q_i, A *if* $G_6(q_i)\left[G_3(m_d, P_m)G_4(m_d, P_m) + \left\{G_6(m_d, P_m)\right\}^2\right] < 0$.

Proof. For proof, see Appendix D and Appendix G.

Proposition 4. *The principal minor of fourth order of the Hessian matrix for the expected profit function is greater than zero at the optimum findings of the decision variables* m_d, K, P_m, q_i, A *if* $G_4(K)G_6(q_i)\left[G_3(m_d, P_m)G_5(m_d, P_m) + \left\{G_8(m_d, P_m)\right\}^2\right] < 0$.

Proof. For proof, see Appendix D and Appendix H.

Proposition 5. *The principal minor of fifth order of the Hessian matrix for the expected profit function is less than zero at the optimum findings of the decision variables* m_d, K, P_m, q_i, A *if* $G_4(K)G_6(q_i)G_7(A)\left[G_3(m_d, P_m)G_5(m_d, P_m) + \left\{G_8(m_d, P_m)\right\}^2\right] < 0$.

Proof. For proof, see Appendix D and Appendix I.

Proposition 6. *The expected profit function is concave at the optimum findings of the decision variables if* $G_3(m_d, P_m) < 0$, $G_3(m_d, P_m)G_5(m_d, P_m) + \left\{G_8(m_d, P_m)\right\}^2 < 0$, $G_6(q_i) > 0$, $G_4(K) > 0$ *and* $G_7(A) > 0$.

Proof. For proof, see Appendix D and Appendix E - Appendix I.

5. Numerical experiment

Managing the overstock situation and increasing sales has become a significant task for every industry. This study designates an SCM under the trade-credit and barter exchange policy. The manufacturer considers flexible production to reach maximum profit despite random demand and improves product quality to maintain product reputation in the world market.

The parametric data are taken from Dey et al. (2021b), Hua et al. (2020), and Wang et al. (2021) at their best fit to find the maximum profit in respect of optimal credit period, production rate, credit period dependent order, quality improvement, and several investments.

The parametric data for the manufacturer are assumed as follows: $I_p = \$1000$, $B_q = \$600$, $\omega = 0.75$, $C_{sme} = \$400$/setup, $C_{ome} = \$200$/order, $U_b = 1000$, $\alpha_1 = 0.02$, $\alpha_{cm} = 0.12$, $\alpha_{3q} = 400$, $g_l = \$400.8$, $\theta_{1p} = 2.25$, $m_w = \$75$/unit, $C_{hmne} = \$0.04$/unit/unit time, $C_{1m} = \$0.12$/unit, $I_{cmp} = 0.6$ unit, $I_{gmp} = 0.4$ unit, $b_m = \$0.03$/unit, $B_{rme}^P = \$14$/unit, $L_{im} = \$1400$/production, $\alpha_{im} = \$0.0006$ /unit, $\rho = 1.25$, $s_{1m} = \$0.1$/unit. The parametric data for the retailer are assumed as follows: $l = 380$, $\vartheta = 500$, $A_{0r} = \$320$/order, $L_b = 115$, $C_{hrbe} = \$0.05$/unit/unit time, $C_{2r} = \$0.12$/unit, $i_{cre} = 0.5$ unit, $i_{gre} = 0.4$ unit, $a_r = \$0.03$/unit, $\theta_l = 0.05$, $p_r = \$115$/unit, $s_{2r} = \$0.12$/unit, $q_{0r} = 40$ unit, $r = \$0.02$/unit.

The optimum findings of the corresponding decision variables related to this model are obtained as follows: the credit period granted by the manufacturer for the retailer is $m_d^* = 0.22$ years, the manufacturer's production rate is $P_m^* = 612.55$ units/year, the quality improvement is $q_i^* = 0.91$, investment to reduce manufacturer's setup is $K^* = \$149.79$, and investment to reduce retailer's ordering cost is $A^* = \$115$; then, the optimum order size is $q^* = 1120.42$ units. Finally, using the Equation (26), the expected total profit of the model is derived as $J_p^{MRCB}(m_d^*, K^*, P_m^*, q_i^*, A^*) = \$10{,}963.68$.

The most significant scenario observed in this study is ignoring flexible production, barter exchange policy, and several investments, the model is very similar to Kaur (2019) in which the total profit was \$5010.04. Therefore, this study is more acceptable and beneficial for the industry.

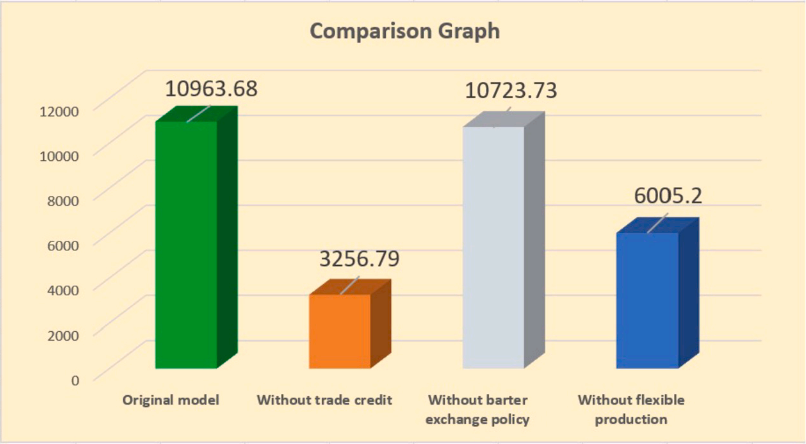

Fig. 3. Comparison of the expected total profit of the original model and special cases.

5.1. Proof of optimality (numerically)

Here, a statistical analysis is done to prove the result conclusively. At the optimal points $\left(m_d^*, P_m^*, K^*, q_i^*, A^*\right) = (0.22, 612.55, 149.79, 0.91, 115)$, the Hessian of the expected joint profit function J_p^{MRCB} is

$$H = \begin{bmatrix} -29571.1 & 0 & 0 & 0 & 0 \\ 0 & -0.02 & 0 & 0 & 0 \\ 0 & 0 & -549.39 & 0 & 0 \\ 0 & 0 & 0 & -0.02 & 0 \\ 0 & 0 & 0 & 0 & -.0001 \end{bmatrix}$$

Clearly, the values of the principle minors at $(0.22, 612.55, 149.79, 0.91, 115)$ are, $|H_{11}|$=-29571.1 < 0, $|H_{22}| = +454.08494 > 0$, $|H_{33}|$=-249471.087 < 0, $|H_{44}| = +4989.07249 > 0$, $|H_{55}|$=-0.498907247 < 0. The principal minors are of opposite signs at the optimal points. Therefore, the expected total profit function J_p^{MRCB} is concave, i.e., maximum at the optimal findings of decision variables.

5.2. Special cases

This section discusses three special cases based on the proposed research. These cases briefly explain why the proposed study is more profitable and acceptable.

5.2.1. Model without trade-credit

This special case elucidates the state of the model after ignoring credit sales; here, the manufacturer does not provide any credit period for the manufacturer. Subsequently, the model transformed to a function of four variables $\left(m_d^*, P_m^*, K^*, q_i^*, A^*\right)$, and the optimal results of the correlating decision variables are P_m^* =612.55 units, q_i^* =0.91, $K^* = \$149.79$, and $A^* = \$115$; then, the optimum order quantity becomes $q_i^* = 950$ units and the expected total profit of the SCM changes to $J_p^{MRCB} = \$3256.79$. This result implies that after neglecting the trade-credit policy, the model's optimal order size and expected total profit decrease. This finding motivates the industrial manager to consider the trade-credit options to increase sales and maximize system profitability.

5.2.2. Model without barter exchange policy

This special case clarifies the model's status after ignoring the barter exchange policy. In this case, selecting a barter platform is neglected; the retailer holds the excess products with a certain holding cost after the sale. In such a situation, the credit period granted from the manufacturer for the retailer is $m_d^* = 0.16$ years, the manufacturer's production rate is P_m^* =612.55 units, the quality improvement is $q_i^* = 0.91$, investment for reducing manufacturer's setup expenditure is $K^* = \$149.79$, investment to reduce retailer's ordering expenditure is $A^* = \$115$, and the maximum profit of the model becomes $J_p^{MRCB}(m_d^*, K^*, P_m^*, q_i^*, A^*) = \$10{,}723.73$. This result exhibits that after neglecting the barter exchange policy, the overall profit of the SCM decreases. This observation is expected to motivate retailers to consider a barter exchange policy to sell unsold products in exchange for the subsidiary products they need and reach the maximum profit levels.

5.2.3. Model without flexible production

This special scenario elaborates the model's status after ignoring the flexible production. In this scenario, if a fixed production rate is considered, the SCM is converted as a function of four variables $\left(m_d^*, K^*, q_i^*, A^*\right)$, and the optimal results of the correlating decision variables are $m_d^* = 0.54$ years, $q_i^* = 0.91$, $K^* = \$149.79$, and $A^* = \$115$, and the expected total profit of the SCM becomes $J_p^{MRCB}(m_d^*, K^*, q_i^*, A^*) = \$6005.20$. This result indicates that neglecting flexible production reduces the system's overall profitability. This investigation is expected to inspire industries to consider flexible production to meet uncertain demand and maximize profits.

Thus, trade-credit, barter exchange policy, and flexible production makes SCM more realistic and profitable and validates the model.

5.2.4. Discussions

Fig. 3 demonstrates the expected total profit of the SCM under several special cases at a glance. From the special cases and Fig. 3, it is found that the system's expected total profit is maximum in the original model. The expected total profit of the original model and its special cases are

Table 3
Sensitivity investigation table.

Parameters	Changes in inputs (%)	Changes in J_p^{MRCB} (%)	Parameters	Changes in inputs (%	Changes in J_p^{MRCB} (%)
C_{ome}	−50	+0.85	C_{sme}	−50	+1.69
	−25	+0.42		−25	+0.85
	+25	−0.42		+25	−0.85
	+50	−0.85		+50	−1.69
C_{hmne}	−50	+0.31	α_{3q}	−50	+1.27
	−25	+0.15		−25	+0.44
	+25	−0.15		+25	−0.28
	+50	−0.31		+50	−0.48
g_l	−50	+0.47	C_{1m}	−50	+0.45
	−25	+0.15		−25	+0.23
	+25	+0.001		+25	−0.23
	+50	+0.15		+50	−0.45
b_m	−50	+0.011	L_{im}	−50	+25.59
	−25	+0.006		−25	11.62
	+25	−0.006		+25	−10.09
	+50	−0.011		+50	−19.10
α_{im}	−50	21.11	C_{hrbe}	−50	+0.35
	−25	+9.41		−25	+0.18
	+25	−8.00		+25	−0.18
	+50	−15.01		+50	−0.55
A_{0r}	−50	+5.87	C_{2r}	−50	+0.45
	−25	+2.44		−25	+0.23
	+25	−1.89		+25	−0.23
	+50	−3.43		+50	−0.45
a_r	−50	+0.003	B^P_{rme}	−50	+162.28
	−25	+0.001		−25	+72.34
	+25	−0.001		+25	−60.94
	+50	−0.003		+50	−113.89

N.F. indicates not feasible.

acquired numerically using MATLAB 2015a software. The proposed model is the modification of such special scenarios discussed here. From special case 1, one can observe that the expected profit decreases in an SCM without a trade-credit policy. Thus industries should adopt such strategies to increase order volume and maximize profits. Next, from special case 2, it is confirmed that the barter exchange policy is beneficial in exchanging the overstock inventory for required merchandise and reaching the maximum profit level. Again, the demand for any product is not always the same, sometimes fluctuating due to quality, service, selling price, or supply. Flexible production helps industries survive in competitive markets and manage demand variability in such situations. From special case 3, it is understood that, for a fixed production rate, the expected profit of the system decreases. Consequently, these special scenarios help industries accept the original case confidently.

6. Sensitivity analysis

Table 3 displays the changes in expected total profit when different input cost parameters are varied by -50%, -25%, +25%, and +50%.

Table 3, displays that the joint profit increases or decreases according to each cost parameter (except g_l) decreases or increases. The findings are discussed as follows:

1. From Table 3, it is evident that if the manufacturer's setup cost (C_{sme}), ordering cost (C_{ome}), and coefficient of cost for product quality improvement (α_{3q}) are decreased, the expected total profit is increased. On increasing setup cost, ordering cost, and coefficient of cost for product quality improvement by 25 or 50%, the expected total profit decreases. From Table 3, it is clear that the absolute value of the negative change in expected total profit is equal to the positive change, and these parameters moderately affect the system.
2. Manufacturer's holding cost (C_{hmne}) and retailer's holding cost (C_{hrbe}) have minor effects on the model. For a negative or positive percentage (25% or 50%) changes in this cost parameter, the expected total profit increases or decreases. Additionally, the results converge in magnitude, i.e., for both changes, equivalent conditions are obtained.
3. The marginal reduction cost b_m of the manufacturer and a_r of the retailer are less sensitive parameters of the SCM. For a negative or positive percentage (25% or 50%) change in this cost parameters, the expected total profit increases or decreases. Additionally, the results match in magnitude, i.e., for both changes, equivalent conditions are obtained.
4. The expected total profit increases or decreases according to the manufacturer's and retailer's carbon costs (C_{1m} and C_{2r}), decreases or increases by 50% or 25%. The expected total profit maintains a position of equilibrium and these parameters moderately affect the system.
5. From Table 3, one can observe that if the manufacturer's tool/die cost (α_{im}), development cost (L_{im}), and the retailer's initial ordering cost (A_{0r}) are increased or decreased by 25% or 50%, the expected total profit is decreased or increased. These are highly sensitive parameters that affect SCM profitability at the highest level.
6. The manufacturer's per unit raw material cost (B^P_{rme}) is the first and foremost sensitive cost parameter for the proposed SCM. Table 3 indicates that if the cost parameter (B^P_{rme}) decreases by 25% or 50%, the expected total profit increases by 72.34% or 162.78% and if (B^P_{rme})) is increased by 25 or 50%, the expected profit is decreased by 60.94 or 113.89%.

Figs. 4–8 illustrate the change in expected total profit with respect to different cost parameters of the model.

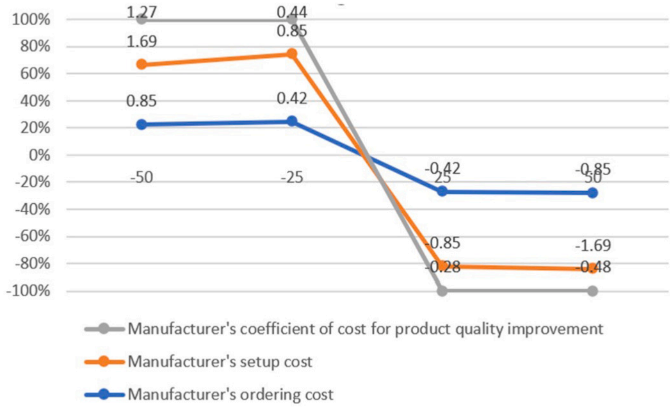

Fig. 4. Sensitivity of the expected total profit concerning different cost parameters.

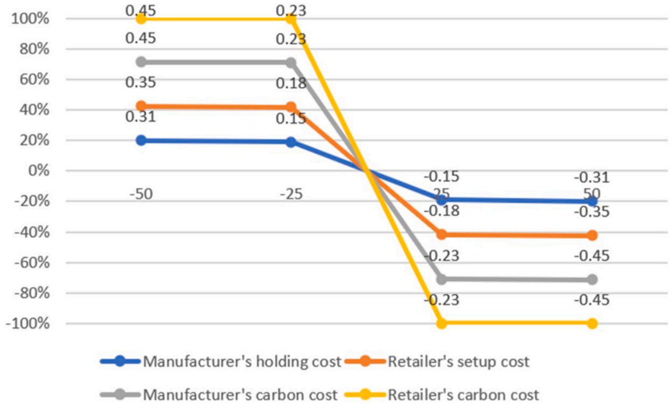

Fig. 5. Sensitivity of the expected total profit concerning different cost parameters.

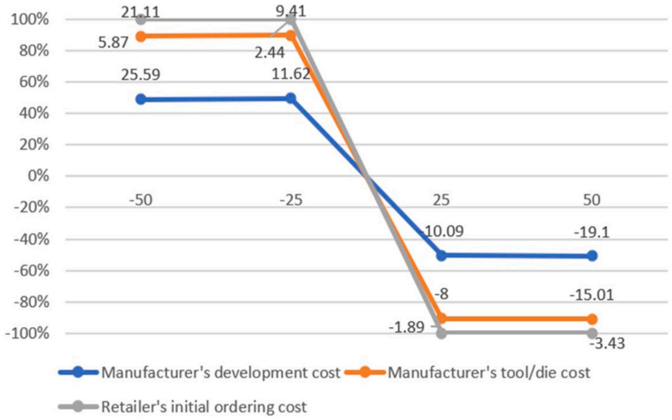

Fig. 6. Sensitivity of the expected total profit concerning different cost parameters.

7. Comparative studies

Table 4 demonstrates the comparison of the present study with some previous studies done in this field.

8. Managerial insights

Business managers can maximize profits through several ideas and technologies. Every industry needs to pay special attention to the scientific observance and statistical data whenever starting and growing a business. In this section, some of the recommendations from this article are mentioned and discussed one by one.

1. A barter exchange is one of the most important principles for every business industry. It is an alternative transaction strategy through which products or services are directly altered for each other without utilizing money as intermediaries. This policy is beneficial for every business industry to move overstock inventory, manage uncertain demand, and reach optimum profit level. Therefore, business managers should focus on this policy.

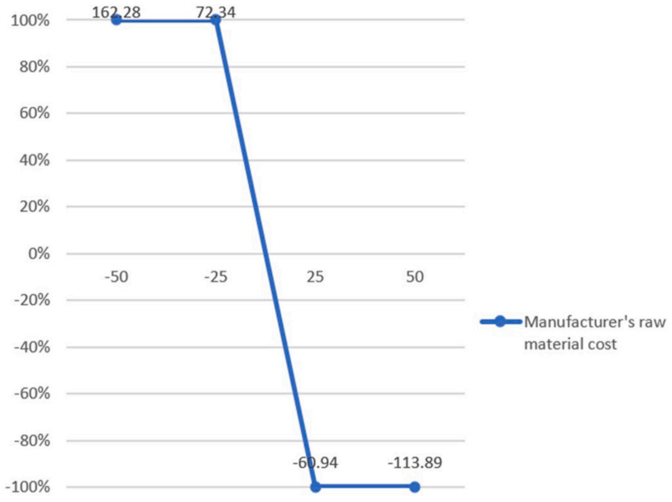

Fig. 7. Sensitivity of the expected total profit concerning different cost parameters.

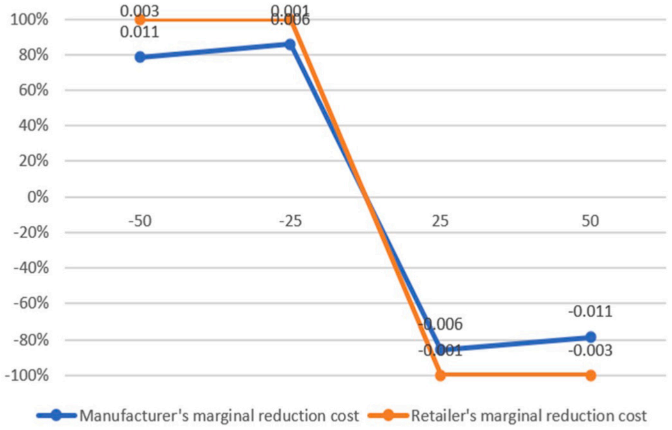

Fig. 8. Sensitivity of the expected total profit concerning different cost parameters.

Table 4
Comparative studies.

	Noh et al. (2019)	Chang et al. (2019)	Kaur (2019)	This study
Demand	variable	Variable	Stochastic	Stochastic
Trade-credit	NC	Considered	Considered	Considered
Production rate	Flexible	Fixed	NC	Flexible
Investment	NA	NA	NA	Done for QI, SCR, OCR
Barter exchange policy	NC	NC	NC	Considered
Total profit	\$10, 528.95	\$7282.53	\$5010.04	\$10, 963.68

NA - Not applicable, NC - Not considered, QI - Quality improvement, SCR- Setup cost reduction, OCR - Ordering cost reduction.

2. Another significant issue discussed in this article is trade-credit. It is a delayed payment agreement that any company provides to its consumers to pay the outstanding amount within this period. The company does not charge any interest during this period. With this trade-credit policy, any company can get more orders, increase sales, discover new customers, and maximize profits. Thus, business managers should provide this policy.
3. Flexible production plays an important role in every industry in handling uncertain demand. Through flexible production, the industry can promote itself in a competitive market despite facing an unexpected demand, reassure consumers about product availability, and manage overstock/understock situations. Therefore, every production company needs to focus on flexible production to deal with such problems and meet the expected profit. In this study, production expenditure per unit is apprehended as an expression of production rate, development expenditure, and die/tool expenditure.
4. Setup cost and ordering cost play a significant role in any SCM. Making some investments to reduce setup and ordering costs is crucial to maximizing profitability in any SCM. In the proposed SCM, the manufacturer makes certain discrete investments to lessen setup costs and the retailer makes a continuous investment to facilitate the ordering costs. Through such investments, business industries can reduce total costs and maximize the total profits of the system. Thus, business managers are advised to make such investments for maximizing the overall profitability of SCM.

5. Again, the success of any business depends on the quality of its products. Therefore, improving product quality is an important issue for every industry. In this study, the manufacturer applies certain investments to improve the product quality of the system. Thus, enterprises are advised to make such investments to enhance the quality of products. With this kind of investment, any company can maintain its product quality, increase sales, and maintain its brand image in the global market.
6. Another important issue that every industry should maintain is environmental sustainability. Large amounts of carbon are discharged for production and inventory, which adversely affects the environment. In this study, an emissions target has been set for each product for the marginal reduction of carbon discharges. Industry supervisors can alleviate the carbon discharges and maintain environmental sustainability with this idea. Therefore, this concept makes the model more realistic from an ecological aspect, and hence every industry should concentrate on it.

Thus, through this proposed work, industry supervisors can make important decisions regarding barter exchange policy, trade-credit, production type, product quality improvement, reduction of setup, ordering, and carbon discharges, which directly increase the expected profit of the entire system.

9. Conclusions

Nowadays, a growing problem in several industries is the overstock situation which is mainly caused due to the uncertainty of demand. Therefore, how industry supervisors would manage such cases and reach the maximum profit level has become an important task. The proposed work solved this issue by considering the barter exchange policy from the retailer’s side to exchange the overstock products at the almost full retail price for the products it needs. This concept motivated the retailer to place more orders as the overstock products could be altered in the barter platform. In this study, the manufacturer approved a trade-credit policy to get maximum charges and reach the top profit level. This model considered flexible production to manage demand uncertainty and avoided overstock and understock situations. The proposed model considered one discrete investment to diminish setup costs and two continuous investments to lessen ordering costs and enhance product quality. This model paid special attention to carbon emission reduction as an environmental issue. The total profit was maximized for the credit period, flexible production, quality improvement, and several investments for ordering and setup cost reduction. MATLAB 2015a software displayed the numerical results and demonstrated global optimization of the profit function. The current work showed that total profit can be increased by 4.13% considering the credit period. However, considering flexible production, barter exchange policy, and several investments in this SCM for stochastic demand, the joint profit was increased up to 50.55%.

The main limitations of this study were two-echelon SCM, trade-credit policy from the manufacturer’s side, and barter exchange policy from the retailer’s side. The proposed study developed a two-echelon SCM. One can extend this work by appraising three/more echelon SCM (Liu et al., 2024). Again, the proposed two-echelon SCM with a single manufacturer and a single retailer can be transformed into an SCM with a single manufacturer multi-retailer (Zhong et al., 2022). Here, the manufacturer offered a delay payment facility to the retailer; therefore, the credit payment advantage from the retailer to the customer can be considered as an extension of the work (Li et al., 2021). Exploring the Stackelberg game strategy to solve the model will be an interesting research aspect (Khanlarzade and Farughi, 2024). This study investigated an SCM in which the unsold products were exchanged by the retailer in a barter platform. However, the manufacturer may sometimes have excess raw materials or finished products due to supply or demand uncertainties. One can extend this study by considering the exchange policy on behalf of the manufacturer. Next, the current two-echelon SCM involving a single manufacturer and single retailer can be transformed into a multi-echelon SCM with a barter exchange policy. For example, in a three-layer SCM involving a single supplier, retailer, and manufacturer, both retailer and manufacturer can engage in a barter platform. Moreover, the autonomation policy for supplying error-free products and utilizing renewable resources to reduce EC can be a further extension of this study (Kugele and Sarkar, 2023). Furthermore, a sustainable SCM with manufacturing, remanufacturing, and improved service facilities may be an exciting future extension of the research (Sarkar and Bhuniya, 2022).

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data is provided in the paper.

Appendix A

Abbreviations

Abbreviations used in this article are shown below:
SCM - supply chain management
CR - continuous-review
EC - energy consumption
RFID - radio frequency identification
SSMD - single-setup multi-delivery
CEC - carbon emission control
CAPT - cap-and-trade
SCR - setup cost reduction
OCR - ordering cost reduction
QI - quality improvement
PPR - production process reliability

REM - remanufacturing
BFP - biofuel production
ATN - autonomation
FC - fuel consumption
GT - green technology
RE - renewable energy

Appendix B

$$
\begin{aligned}
&E(q-x)^{+} \leq \frac{1}{2}\left[\sqrt{\vartheta^{2}+(q-l)^{2}}-(l-q)\right], \\
&E(x-q)^{+} \leq \frac{1}{2}\left[\sqrt{\vartheta^{2}+(q-l)^{2}}-(q-l)\right], \\
&E((q-q_{0r})-x)^{+} \leq \frac{1}{2}\left[\sqrt{\vartheta^{2}+((q-q_{0r})-l)^{2}}-(l-(q-q_{0r}))\right], \\
&E(x-(q-q_{0r}))^{+} \leq \frac{1}{2}\left[\sqrt{\vartheta^{2}+((q-q_{0r})-l)^{2}}-((q-q_{0r})-l)\right], \\
&G_1(m_d,P_m)= \\
&-\Bigg[\left\{C_{1m}I_{gmp}+C_{2r}I_{gre}+b_m(I_{cmp}-I_{gmp})^2+a_r(I_{cre}-I_{gre})^2+B_{rme}^{P}+L_{im}/P_m+\alpha_{im}P_m^{\rho}\right\} \\
&+\frac{C_{hmne}+C_{hrbe}+rp_r-2p_r-s_{1m}-s_{2r}}{2}+\frac{C_{hrbe+p_r}}{2}\left\{\frac{B_q e^{\omega m_d}-q_{0r}-l}{\sqrt{\vartheta^{2}+(B_q e^{\omega m_d}-q_{0r}-l)^{2}}}\right\}\Bigg], \\
&G_2(m_d)=\frac{p_r+s_{1m}+s_{2r}+C_{hmne}+rp_r}{2\sqrt{\vartheta^{2}+(B_q e^{\omega m_d}-l)^{2}}}.
\end{aligned}
\tag{B.1}
$$

Appendix C

Differentiating Equation (26) partially with respect to the decision variables m_d, K, P_m, A and q_i, one can obtain

$$
\begin{aligned}
&\frac{\partial J_p MRCB}{\partial m_d}=B_q\omega e^{\omega m_d}\left\{G_1(m_d,P_m)-G_2(m_d)(B_q e^{\omega m_d}-l)\right\} \\
&\frac{\partial J_p MRCB}{\partial K}=U_b\alpha_1 e^{-\alpha_1 K}-1 \\
&\frac{\partial J_p MRCB}{\partial P_m}=-B_q e^{\omega m_d}\left\{-\frac{L_{im}}{P_m^2}+\alpha_{im}\rho P_m^{\rho-1}\right\} \\
&\frac{\partial J_p MRCB}{\partial A}=\frac{L_b}{A}-1 \\
&\frac{\partial J_p MRCB}{\partial q_i}=g_l-\frac{\alpha_{3q}\theta_{1p}q_i^{\theta_{1p}-1}}{2}.
\end{aligned}
$$

Appendix D

Partial derivatives of the second order

The second order partial derivatives of the joint profit function are as follows:

$$
\begin{aligned}
&\frac{\partial^2 J_p^{MRCB}}{\partial m_d^2}=B_q\omega^2 e^{\omega m_d}\left\{G_1(m_d,P_m)-G_2(m_d)(B_q e^{\omega m_d}-1)\right\}-\frac{C_{hrbe}+p_r}{2}\left[\frac{\left(B_q\vartheta\omega e^{\omega m_d}\right)^2}{\left\{\vartheta^2+(B_q e^{\omega m_d}-l)^2\right\}^{\frac{3}{2}}}\right]-G_2(m_d)\left[\frac{\left(B_q\vartheta\omega e^{\omega m_d}\right)^2}{\vartheta^2+(B_q e^{\omega m_d}-l)^2}\right]=G_3(m_d,P_m) \\
&\frac{\partial^2 J_p^{MRCB}}{\partial K^2}=-U_b\alpha_1^2 e^{-\alpha_1 K}=-G_4(K) \\
&\frac{\partial^2 J_p^{MRCB}}{\partial P_m^2}=-B_q e^{\omega m}\left[\frac{2L_{im}}{P_m^3}+\alpha_i\rho\left(\rho-1\right)P_m^{\rho-2}\right]=-G_5(m_d,P_m) \\
&\frac{\partial^2 J_p^{MRCB}}{\partial q_i^2}=-\frac{\alpha_3\theta_1\left(\theta_1-1\right)}{2}q_i^{\theta_1-2}=-G_6(q_i) \\
&\frac{\partial^2 J_p^{MRCB}}{\partial A^2}=-\frac{L_b}{A^2}=-G_7(A) \\
&\frac{\partial^2 J_p^{MRCB}}{\partial m_d\partial P_m}=-B_q\omega e^{\omega m_d}\left\{-\frac{L_i}{P_m^2}+\alpha_i\rho P_m^{\rho-1}\right\}=-G_8(m_d,P_m)
\end{aligned}
$$

$$\frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial q_i} = \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial K} = \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial A} = \frac{\partial^2 J_p^{MRCB}}{\partial q_i \partial P_m} = \frac{\partial^2 J_p^{MRCB}}{\partial q_i \partial K} = \frac{\partial^2 J_p^{MRCB}}{\partial q_i \partial A} = \frac{\partial^2 J_p^{MRCB}}{\partial P_m \partial K} = \frac{\partial^2 J_p^{MRCB}}{\partial P_m \partial A} = \frac{\partial^2 J_p^{MRCB}}{\partial A \partial K} = 0$$

where, $G_3(m_d, P_m) = B_q \omega^2 e^{\omega m_d} \left\{ G_1(m_d, P_m) - G_2(m_d)(B_q e^{\omega m_d} - 1) \right\} - \frac{C_{hrbe} + p_r}{2} \left[\frac{\left(B_q \vartheta \omega e^{\omega m_d}\right)^2}{\left\{ \vartheta^2 + (B_q e^{\omega m_d} - l)^2 \right\}^{\frac{3}{2}}} \right] - G_2(m_d) \left[\frac{\left(B_q \vartheta \omega e^{\omega m_d}\right)^2}{\vartheta^2 + (B_q e^{\omega m_d} - l)^2} \right]$

$G_4(K) = U_b \alpha_1^2 e^{-\alpha_1 K}$

$G_5(m_d, P_m) = B_q e^{\omega m_d} \left[\frac{2L_{im}}{P_m^3} + \alpha_i \rho (\rho - 1) P_m^{\rho - 2} \right]$

$G_6(q_i) = \frac{\alpha_3 \theta_1 (\theta_1 - 1)}{2} q_i^{\theta_1 - 2}$

$G_7(A) = \frac{L_b}{A^2}$

$G_8(m_d, P_m) = B_q \omega e^{\omega m_d} \left\{ -\frac{L_{im}}{P_m^2} + \alpha_i \rho P_m^{\rho - 1} \right\}$.

Different principal minors

The Hessian matrix (H) at the optimum points is discussed below.

$$H = \begin{bmatrix} \frac{\partial^2 J_p^{MRCB}}{\partial m_d^2} & \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial P_m} & \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial q_i} & \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial K} & \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial A} \\ \frac{\partial^2 J_p^{MRCB}}{\partial P_m \partial m_d} & \frac{\partial^2 J_p^{MRCB}}{\partial P_m^2} & \frac{\partial^2 J_p^{MRCB}}{\partial P_m \partial q_i} & \frac{\partial^2 J_p^{MRCB}}{\partial P_m \partial K} & \frac{\partial^2 J_p^{MRCB}}{\partial P_m \partial A} \\ \frac{\partial^2 J_p^{MRCB}}{\partial q_i \partial m_d} & \frac{\partial^2 J_p^{MRCB}}{\partial q_i \partial P_m} & \frac{\partial^2 J_p^{MRCB}}{\partial q_i^2} & \frac{\partial^2 J_p^{MRCB}}{\partial q_i \partial K} & \frac{\partial^2 J_p^{MRCB}}{\partial q_i \partial A} \\ \frac{\partial^2 J_p^{MRCB}}{\partial K \partial m_d} & \frac{\partial^2 J_p^{MRCB}}{\partial K \partial P_m} & \frac{\partial^2 J_p^{MRCB}}{\partial K \partial q_i} & \frac{\partial^2 J_p^{MRCB}}{\partial K^2} & \frac{\partial^2 J_p^{MRCB}}{\partial K \partial A} \\ \frac{\partial^2 J_p^{MRCB}}{\partial A \partial m_d} & \frac{\partial^2 J_p^{MRCB}}{\partial A \partial P_m} & \frac{\partial^2 J_p^{MRCB}}{\partial A \partial q_i} & \frac{\partial^2 J_p^{MRCB}}{\partial A \partial K} & \frac{\partial^2 J_p^{MRCB}}{\partial A^2} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 J_p^{MRCB}}{\partial m_d^2} & \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial P_m} & 0 & 0 & 0 \\ \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial P_m} & \frac{\partial^2 J_p^{MRCB}}{\partial P_m^2} & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial^2 J_p^{MRCB}}{\partial q_i^2} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial^2 J_p^{MRCB}}{\partial K^2} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial^2 J_p^{MRCB}}{\partial A^2} \end{bmatrix}$$

Appendix E

The principal minor of first order is

$$|\ H_{11}\ | = \frac{\partial^2 J_p^{MRCB}}{\partial m_d^2} = G_3(m_d, P_m).$$

Appendix F

The principal minor of second order is

$$|\ H_{22}\ | = \begin{vmatrix} \frac{\partial^2 J_p^{MRCB}}{\partial m_d^2} & \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial P_m} \\ \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial P_m} & \frac{\partial^2 J_p^{MRCB}}{\partial P_m^2} \end{vmatrix} = \frac{\partial^2 J_p^{MRCB}}{\partial m_d^2} \frac{\partial^2 J_p^{MRCB}}{\partial P_m^2} - \left\{ \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial P_m} \right\}^2 = -[G_3(m_d, P_m) G_5(m_d, P_m) + \left\{ G_8(m_d, P_m) \right\}^2].$$

Appendix G

The principal minor of third order is

$$|\ H_{33}\ | = \begin{vmatrix} \frac{\partial^2 J_p^{MRCB}}{\partial m_d^2} & \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial P_m} & 0 \\ \frac{\partial^2 J_p^{MRC}}{\partial m_d \partial P_m} & \frac{\partial^2 J_p^{MRCB}}{\partial P_m^2} & 0 \\ 0 & 0 & \frac{\partial^2 J_p^{MRCB}}{\partial q_i^2} \end{vmatrix} = \frac{\partial^2 J_p^{MRCB}}{\partial m_d^2} \frac{\partial^2 J_p^{MRCB}}{\partial P_m^2} \frac{\partial^2 J_p^{MRCB}}{\partial q_i^2} - \left\{ \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial P_m} \right\}^2 \frac{\partial^2 J_p^{MRCB}}{\partial q_i^2}$$

$$= G_6(q_i)[G_3(m_d, P_m) G_5(m_d, P_m) + \left\{ G_8(m_d, P_m) \right\}^2].$$

Appendix H

The principal minor of fourth order is

$$|\ H_{44}\ | = \begin{vmatrix} \frac{\partial^2 J_p^{MRCB}}{\partial m_d^2} & \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial P_m} & 0 & 0 \\ \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial P_m} & \frac{\partial^2 J_p^{MRCB}}{\partial P_m^2} & 0 & 0 \\ 0 & 0 & \frac{\partial^2 J_p^{MRCB}}{\partial q_i^2} & 0 \\ 0 & 0 & 0 & \frac{\partial^2 J_p^{MRCB}}{\partial K^2} \end{vmatrix}$$

$$= \frac{\partial^2 J_p^{MRCB}}{\partial m_d^2} \frac{\partial^2 J_p^{MRCB}}{\partial P_m^2} \frac{\partial^2 J_p^{MRCB}}{\partial q_i^2} \frac{\partial^2 J_p^{MRCB}}{\partial K^2} - \left\{ \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial P_m} \right\}^2 \frac{\partial^2 J_p^{MRCB}}{\partial q_i^2} \frac{\partial^2 J_p^{MRCB}}{\partial K^2}$$

$$= -G_4(K) G_6(q_i)[G_3(m_d, P_m) G_5(m_d, P_m) + \left\{ G_8(m_d, P_m) \right\}^2].$$

Appendix I

The principal minor of fifth order is

$$
\begin{aligned}
\mid H_{55} \mid &= \begin{vmatrix}
\frac{\partial^2 J_p^{MRCB}}{\partial m_d^2} & \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial P_m} & 0 & 0 & 0 \\
\frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial P_m} & \frac{\partial^2 J_p^{MRCB}}{\partial P_m^2} & 0 & 0 & 0 \\
0 & 0 & \frac{\partial^2 J_p^{MRCB}}{\partial q_i^2} & 0 & 0 \\
0 & 0 & 0 & \frac{\partial^2 J_p^{MRCB}}{\partial K^2} & 0 \\
0 & 0 & 0 & 0 & \frac{\partial^2 J_p^{MRCB}}{\partial K^2}
\end{vmatrix} \\
&= \frac{\partial^2 J_p^{MRCB}}{\partial m_d^2} \frac{\partial^2 J_p^{MRCB}}{\partial P_m^2} \frac{\partial^2 J_p^{MRCB}}{\partial q_i^2} \frac{\partial^2 J_p^{MRCB}}{\partial K^2} \frac{\partial^2 J_p^{MRCB}}{\partial A^2} - \left\{ \frac{\partial^2 J_p^{MRCB}}{\partial m_d \partial P_m} \right\}^2 \frac{\partial^2 J_p^{MRCB}}{\partial q_i^2} \frac{\partial^2 J_p^{MRCB}}{\partial K^2} \frac{\partial^2 J_p^{MRCB}}{\partial A^2} \\
&= G_4(K) G_6(q_i) G_7(A) [G_3(m_d, P_m) G_5(m_d, P_m) + \left\{ G_8(m_d, P_m) \right\}^2].
\end{aligned}
$$

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